2 PATH INTEGRAL FOR A SCALAR FIELD THEORY

2.1 Formal definition of QFT

→ We consider the theory defined by the path integral

$$\int \mathcal{D}\varphi(x) \exp\{ i S[\varphi] \}$$

We want to compute correlation functions of operators.

→ We start with a simple example: let's compute

$$\int \mathcal{D}\varphi \; \varphi(x_1)\varphi(x_2) \exp\{ i \int_{-T}^{T} d^4x \, \mathcal{L}[\varphi] \}$$

with boundary conditions:

$$\varphi(-T,\bar{x}) = \varphi_{in}(\bar{x}) \text{ and } \varphi(T,\bar{x}) = \varphi_f(\bar{x}).$$

↳ We suppose $-T < x_0^1 < x_0^2 < T$ and we call $\varphi(x_0^1,\bar{x}) = \varphi_1(\bar{x})$ and $\varphi(x_0^2,\bar{x}) = \varphi_2(\bar{x})$. We then split the path integral:

$$\int \mathcal{D}\varphi \exp\{ i \int_{-T}^{x_1^0} \mathcal{L}\, d^4x \} \varphi_1(\bar{x}) \int \mathcal{D}\varphi \exp\{ i \int_{x_1^0}^{x_2^0} \mathcal{L}\, d^4x \} \varphi_2(\bar{x}) \int \mathcal{D}\varphi \, e^{i\int_{x_2^0}^{T} \mathcal{L} d^4x}$$

$$= \langle \varphi_f(\bar{x}) | U(T,x_2^0) \hat{\varphi}(\bar{x}_2) U(x_2^0,x_1^0) \hat{\varphi}(\bar{x}_1) U(x_1^0,-T) | \varphi_{in}(\bar{x}) \rangle$$

↳ We use Heisenberg picture operators by writing

$$\hat{\varphi}(t,\bar{x}) = U(0,t)\hat{\varphi}(\bar{x}) U(t,0)$$

Then,

$$\int \mathcal{D}\varphi \cdot \varphi_1(x)\varphi_2(x) e^{iS} = \langle \varphi_f | \underbrace{e^{-iHt}}_{U(T,0)} \mathcal{T}[\varphi(x_1)\varphi(x_2)] \underbrace{e^{-iHt}}_{U(0,-T)} | \varphi_{in} \rangle$$

⊙ Down with boundary conditions!

→ To get a Poincaré invariant quantity, we need to push the conditions to ± infinity time. Let's proceed:

$$\rightarrow e^{-iHT}|\varphi_{in}\rangle = \sum_{|n\rangle} e^{-iE_nT} |n\rangle\langle n|\varphi_{in}\rangle$$

$$= e^{-iE_0T} |0\rangle\langle 0|\varphi_i\rangle + \sum_{n\neq 0} e^{-iE_nT} |n\rangle\langle n|\varphi_{in}\rangle$$

$$= e^{-iE_0T}\left(|0\rangle\langle 0|\varphi_i\rangle + \sum_{n\neq 0} e^{-i(E_n-E_0)T} |n\rangle\langle n|\varphi_{in}\rangle \right.$$

By taking $T \to \infty(1-i\epsilon)$, the term $e^{-i(E_n-E_0)T} \to 0$

The imaginary component of the limit is to make sure the expon. is at least partly real, to get $\lim_{x\to\infty} e^{-x} = 0$. We're left with

$$e^{-iHT}|\varphi_i\rangle = e^{-iE_0T}|0\rangle\langle 0|\varphi_i\rangle.$$

$$\hookrightarrow \lim_{T\to\infty(1-i\epsilon)} \int \mathcal{D}\varphi\,(\varphi(x_1)\varphi(x_2)\dots)\,e^{i\int_{-T}^{T} d^4x\,\mathcal{L}} = \left(e^{-2iE_0T}\langle 0|\varphi_{in}\rangle\langle\varphi_f|0\rangle\right) \times \langle 0|T[\varphi(x_1)\varphi(x_2)\dots]|0\rangle$$

It's the product of path integral with and without insertion. By normalizing, we get:

DEF The formal definition of QFT can be taken as

$$\langle 0|T[\hat{O}_1(x_1)\hat{O}_2(x_2)\dots]|0\rangle = \frac{\int\mathcal{D}\varphi\,(O_1(x_1)O_2(x_2)\dots)\,e^{i\int d^4x\,\mathcal{L}}}{\int\mathcal{D}\varphi\,e^{i\int d^4x\,\mathcal{L}}}$$

We can now derive propagators and Feynman rules from this path integral definition of QFT. We'll start from a free theory, and introduce interactions as perturbation of the free theory.

2.2 Free real scalar theory

→ We start from $\int\mathcal{D}\varphi\,(\varphi(x_1)\dots)\exp\{i\int d^4x\left(\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2\right)\}$

Since the phase is quadratic in φ, the path integral is essentially a Gaussian integral

⊙ Toy-model:

→ To warm up, let's start with:

$$\int dv_i \exp\{-v_i K_{ij} v_j\} = \int dv_i \exp\{-v'^T \overbrace{O^T K O}^{K'} v'\}$$

$$= \int dv_i' \exp\{\textstyle\sum_i k_i (v_i')^2\} = \prod_{i=1}^{n}\frac{\sqrt{\pi}}{\sqrt{k_i}} = \sqrt{\frac{\pi^n}{\det K}}$$

↳ With insertions, we get:

$$\int dv_i \, v_k v_\ell \exp\{-v_i K_{ij} v_j\} = \int dv_i' \, M_{km} v_m' M_{\ell n} v_n' e^{-\sum_i k_i |v_i|^2}$$

$\frac{1}{2k_m}$?

$$= M_{km} M_{\ell n} \delta_{mn} \prod_{i \neq m} \sqrt{\frac{\pi}{k_i}} \cdot \frac{1}{2k_m} \sqrt{\frac{\pi}{k_m}}$$

⚠ $\int dv . v . e^{-v^2} = 0 \Rightarrow \delta_{m,n}$

$$= \sum_m \frac{M_{km} M_{\ell m}}{2k_m} \sqrt{\frac{\pi^n}{\det K}} = \frac{1}{2} \sqrt{\frac{\pi^n}{\det K}} (K^{-1})_{k\ell}$$

p290 Peskin ⊙ **Generating function of correlators:**

DEF We consider a source $J(x)$ in our action, and we define the generating functional $Z[J]$ as follow:

$$Z[J] \equiv \int \mathcal{D}\varphi \exp\Big[i \int d^4x \{\mathcal{L} + \underbrace{J(x)\varphi(x)}_{\text{source term}}\}\Big]$$

and the functional derivative $\delta / \delta J(x)$ as follows:

$$\frac{\delta}{\delta J(x)} J(y) = \delta^{(4)}(x-y) \Leftrightarrow \frac{\delta}{\delta J(x)} \int d^4x \, J(y)\varphi(y) = \varphi(x)$$

→ We have $\frac{\delta}{\delta J(x)} \exp\Big[i \int d^4y \, J(y)\varphi(y)\Big] = i\varphi(x) \exp\Big[i \int d^4y \, J(y)\varphi(y)\Big]$

$$\Rightarrow \frac{\delta}{\delta i J(x)} Z[J] = \int \mathcal{D}\varphi \, \varphi(x) \exp\Big[i \int d^4x \{\mathcal{L}[\varphi] + J(x)\varphi(x)\}\Big]$$

Correlation function of the K-G field theory can be simply computed by taking functional derivatives of the generative functional

Prop The n-point correlation can be written as:

$$\langle 0 | T[\varphi_1 \varphi_2 \ldots] | 0 \rangle = \frac{1}{Z(0)} \left(\frac{\delta}{\delta i J(x_1)} \cdot \frac{\delta}{\delta i J(x_2)} \cdots \right) Z[J] \Big|_{J=0}$$

⊙ **Feynmann propagator:**

→ $Z[J]$ is a gaussian integral with a shift → it can be computed. Let's start with a simple case:

$$z(s) = \int dv_i \exp\left[-v^T K v - v^T s - s^T v\right] \quad \text{with } s_i \in \mathbb{R}$$

We have:

$$\mathcal{Z}(s) = \int dv_i \exp\left[-(v+K^{-1}s)^T K (v+K^{-1}s) + s^T K^{-1} s\right]$$

$$= e^{s^T K^{-1} s} \int d\tilde{v}_i \exp\left[-\tilde{v}^T \tilde{K} \tilde{v}\right] = e^{s^T K^{-1} s}\, Z[0]$$

→ Similarly, in the case of a free real scalar field, we have:

$$\int d^4x \left(\tfrac{1}{2}\varphi(-\partial^2-m^2)\varphi + J\varphi\right) = \int d^4x \left(\tfrac{1}{2}\left(\varphi + J(-\partial^2-m^2)^{-1}\right)(-\partial^2-m^2)\left(\varphi + (-\partial^2-m^2)^{-1}J\right) - \tfrac{1}{2} J(-\partial^2-m^2)^{-1}J\right)$$

↳ What is the inverse of $-\partial^2-m^2$? It's an operator such that $(-\partial^2-m^2)D = \delta^4(x-y)$

↳ D is a green function! We get:

$$Z[J] = \exp\left[-\tfrac{1}{2}i\int d^4x \int d^4y\, J(x) D(x-y) J(y)\right] \underbrace{\int \mathcal{D}\tilde{\varphi} \exp\left[i\int d^4x\, \tilde{\varphi}(-\partial^2-m^2)\tilde{\varphi}\right]}_{Z[0]}$$

→ By taking the functional δ on this expression of $Z[J]$, we get:

$$\langle 0|T[\varphi_1\varphi_2]|0\rangle = \frac{1}{Z_0}\frac{\delta}{\delta iJ(x)}\frac{\delta}{\delta iJ(y)} Z[J]\Big|_{J=0}$$

$$= iD(x-y) = \frac{i\,\delta^4(x-y)}{-\partial^2-m^2+i\epsilon} = \int \frac{d^4k}{(2\pi)^4}\frac{i\, e^{-ik(x-y)}}{-k^2-m^2+i\epsilon}$$

DEF The Feynman propagator in Fourier space is

$$D_F(x-y) \equiv \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2-m^2+i\epsilon}$$

→ Small recap:

→ We had $Z[J] = \int \mathcal{D}\varphi \exp\left[iS[\varphi] + i\int d^4x\, J(x)\varphi(x)\right]$

free

$$= Z[0]\exp\left[-\tfrac{1}{2}\int d^4x\, J(x) D(x-y) J(y)\right]$$

→ $\langle \varphi_1\varphi_2\rangle = \frac{\delta}{\delta iJ_1}\frac{\delta}{\delta iJ_2}\frac{Z[J]}{Z[0]}\Big| = iD(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{i e^{-ik\cdot(x-y)}}{k^2-m^2+i\epsilon}$

⊙ Higher point functions:

→ E.g. the 4-pt function:

$$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle \equiv \langle 0 | T[\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)] | 0 \rangle$$

$$= \frac{\delta}{\delta iJ_1}\frac{\delta}{\delta iJ_2}\frac{\delta}{\delta iJ_3}\frac{\delta}{\delta iJ_4} \exp\left\{ \frac{-1}{2} i \int J_x D_{xy} J_y \right\}\Big|$$

$$= \frac{\delta}{\delta iJ_1}\frac{\delta}{\delta iJ_2}\frac{\delta}{\delta iJ_3}\left(-\frac{1}{2}\cdot 2 \int D_{x4} J_x \, e^{-i/2 \int J_x D_{xy} J_y} \right)\Big|$$

$$= \frac{\delta}{\delta iJ_1}\frac{\delta}{\delta iJ_2}\left(i D_{34} e^{-i/2 \int J_x D_{xy} J_y} + \int D_{x4} J_x \int D_{y3} J_y \, e^{-i/2 \int J_x D_{xy} J_y} \right)\Big|$$

$$= \frac{\delta}{\delta iJ_1}\left\{ i D_{34}\left(-\int D_{x2} J_x\right) - i D_{24} \int D_{y3} J_y - i D_{23} \int D_{x4} J_4 \right\} e^{-i/2 \int J_x D_{xy} J_y}\Big|$$

$$= iD_{12}\, iD_{34} + iD_{13}\, iD_{24} + iD_{14}\, iD_{23} \rightsquigarrow$$

$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle$
$+$
$+$

Prop 1 The path integral gives automatically all the Wick contractions.

2.3 Real scalar field with interactions

DEF The Lagrangian density for a real scalar field with a quartic interaction is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{1}{4}\lambda \varphi^4 = \mathcal{L}_0 + \mathcal{L}_I$$

where λ is the coupling constant.

→ The path integral of correlation functions now reads:

$$\langle \varphi_1 \cdots \varphi_n \rangle_I = \int \mathcal{D}\varphi \cdot \varphi_1 \cdots \varphi_n \exp\left\{ i\int \mathcal{L}_0 + i \int \mathcal{L}_I \right\}$$

↳ Since $\lambda \ll 1$, $\mathcal{L}_I$ will be treated as a perturbation:

$$\langle \varphi_1 \cdots \varphi_n \rangle_I = \int \mathcal{D}\varphi \cdot \varphi_1 \cdots \varphi_n \left(1 + i \int \mathcal{L}_I + \frac{1}{2}\left(i \int \mathcal{L}_I\right)^2 + \dots \right) e^{i\int \mathcal{L}_0}$$

$$= \langle \varphi_1 \cdots \varphi_n \rangle_{free} + i \underbrace{\left\langle \varphi_1 \cdots \varphi_n \int \mathcal{L}_I \right\rangle_{free}}_{(n+4)\text{ pt function}} + \frac{1}{2} \underbrace{\langle \varphi_1 \cdots \varphi_n \, iS_I \, iS_I \rangle_{free}}_{(n+8)\text{ pt function}} + \dots$$

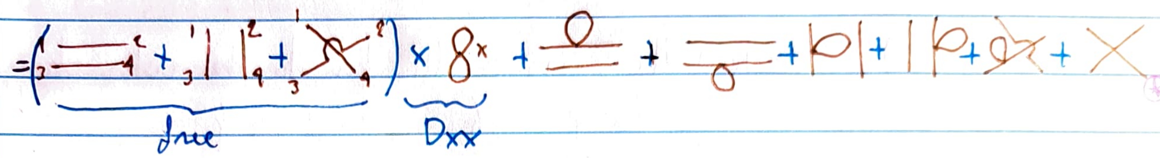
↳ We can write: $\langle \varphi_1 \cdots \varphi_n \rangle_I = \langle \varphi_1 \cdots \varphi_n e^{iS_I} \rangle_{free}$

The correlators in the interacting theory can be computed in the free theory, in expansion in λ

⊙ Feynman rules:

→ Let's compute the 1st to the free result for the 4-pt function:

$\langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle_I = \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle - \frac{i\lambda}{4} \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \int d^4x \, \varphi_x^4 \rangle_{free}$



↳ Only one connected diagram (*):

$-\frac{i\lambda}{4} \cdot 4! \int d^4x \; iD_{1x} \, iD_{2x} \, iD_{3x} \, iD_{4x}$

$= (-6i\lambda) \int_x iD_{1x} \cdots iD_{4x}$ where $4!$ is the combinatorial factor

Prop | The disconnected Feynman diagrams are higher order correction to ones without loops

→ Disconnected graphs with are ill-defined mathematically. Indeed,

$D_{xy} = \frac{1}{\partial_x^2 - m^2} \delta^4(x-y) \xrightarrow{x \to y} \propto \delta^4(x-x)$ (?)

Prop | The complete (exact, physical) propagator will be the one where all correction have been summed

⊙ Connecting diagrams:

→ In order to get rid of disconnected diagrams let us go back to $Z[J]$ in the free theory. The only connected diagram was the 2 pts function propagator, all higher pt function are disconnected.

? → What generate the connect diagram is $\log Z[J]$

DEF (V1) We define $W[J]$ such that $Z[J] = e^{-iW[J]}$

$\Leftrightarrow -iW[J] = \log Z[J]$

! Prop In the free theory, $W[J]$ generates only the connected Green functions

Indeed, let's look at 2 examples:

$\rightarrow \langle \varphi_1 \varphi_2 \rangle = \frac{\delta}{\delta i J_1} \frac{\delta}{\delta i J_2} (-iW[J]) = i D_{12}$

$\rightarrow \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle = \frac{\delta}{\delta i J_1} \frac{\delta}{\delta i J_2} \frac{\delta}{\delta i J_3} \frac{\delta}{\delta i J_3} \left(-i \frac{1}{2} \int J_x D_{xy} J_y\right) = 0$

DEF (V2) We extend the previous definition to the interacting theory:

$$e^{-iW[J]} = \int \mathcal{D}\varphi \exp\{ i S_{tot} + i \int J\varphi \}$$

⊙ Example: the $\lambda\varphi^4$ theory:

→ This theory contains connected higher pt-functions, and has a $\mathbb{Z}_2$ symmetry $\phi \mapsto -\phi$, so that all odd-pt function are 0.

→ Assuming $W[J]$ generates only connected Green functions, we write:

! $$-iW[J] = \frac{1}{2} \int d^4x\, d^4y\, iJ(x)\, J(y) \langle \varphi_x \varphi_y \rangle_c + \frac{1}{4!} \int d^4x_1 \dots d^4x_4\, iJ(x_1) \cdots iJ(x_4) \langle \varphi_{x_1} \varphi_{x_2} \varphi_{x_3} \varphi_{x_4} \rangle_c + \dots$$

We can compute: $Z[J] = e^{-iW[J]} = 1 - iW + \frac{1}{2}(iW)^2 + \dots$

so that:

? $$Z[J] = 1 + \frac{1}{2} \iint iJ_1 iJ_2 \langle \varphi_1 \varphi_2 \rangle_c + \frac{1}{4!} \iiiint iJ_1 \cdots iJ_4 \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle_c + \frac{1}{8} \iiiint iJ_1 \cdots iJ_4 \langle \varphi_1 \varphi_2 \rangle_c \langle \varphi_3 \varphi_4 \rangle_c + \mathcal{O}(J^6)$$

We have: $\frac{\delta}{\delta i J_1} \dots \frac{\delta}{\delta i J_4} Z[J] = \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle$

$$= \langle \varphi_1 \varphi_2 \varphi_3 \varphi_4 \rangle_c + \langle \varphi_1 \varphi_2 \rangle_c \langle \varphi_3 \varphi_4 \rangle_c + \langle \varphi_1 \varphi_3 \rangle_c \langle \varphi_2 \varphi_4 \rangle_c + \langle \varphi_1 \varphi_4 \rangle_c \langle \varphi_2 \varphi_3 \rangle_c$$

It's a sum of connected and disconnect diagrams!

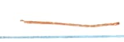
• Removing boring diagrams:

→ We have not removed yet all redundancy from the order by order computation of correlators.
For example, in the $\lambda\varphi^4$ theory, at $\mathcal{O}(\lambda^2)$ propagator, we have:

 + . But the last one is a repetition of 2 $\mathcal{O}(\lambda)$ diagrams. 

DEF We call one-particle irreducible 1PI diagrams wich cannot be split in 2 by cutting an internal propagator.

→ Consider the propagator in $\lambda\varphi^4$:

→ $\mathcal{O}(\lambda^0)$: $\langle \varphi_x \varphi_y \rangle = i D_{xy}$ 

in Fourier space: $iD = \frac{i}{k^2 - m^2}$

→ $\mathcal{O}(\lambda)$: $\langle \varphi_x \varphi_y \rangle = iD(-iM)iD$ 

→ $\mathcal{O}(\lambda^2)$: $\langle \varphi_x \varphi_y \rangle = iD(-iM)iD(-iM)iD$ 

↳ The sequence of non-1PI diagrams can by summarized as:

$$iD_{tot} = iD\sum_{n=0}^{\infty}(-iM)^n(iD)^n = iD\sum_{n=0}^{\infty}(MD)^n \qquad \left(\sum q^n = \frac{1}{1-q}\right)$$

$$= \frac{iD}{1-MD} = \frac{i}{D^{-1}-M} = \frac{i}{k^2-m^2-\mathcal{M}(k^2,m^2)}$$

DEF We define an amputated diagram if it doesn't take into account external lines

DEF The effective action Γ is the generating functional for 1PI:

$$\Gamma \equiv \int \frac{d^4k}{(2\pi)^4}\, \varphi(-k)^* \left(k^2 - m^2 - \mathcal{M}(k^2,m^2)\right)\varphi(k)$$

Before, in Fourier space, we had:

$$S = \int \frac{d^4k}{(2\pi)^4}\, \varphi(-k)^* (k^2-m^2)\,\varphi(k)$$

p 366 Peskin ⓪ **Legendre-transform and classical field:**

→ We consider $e^{-iW[J]} = \int \mathcal{D}\varphi \, e^{iS + i\int J\varphi}$

DEF We define the classical field φ_{cl} by

$$\varphi_{cl}[J] \equiv \frac{\delta}{\delta iJ}(-iW[J]) \equiv \langle\varphi\rangle_J \quad \text{at sources on}$$

Inverting the relation $\varphi_{cl}[J]$, we can compute $J[\varphi_{cl}]$

DEF (V2) We define the effective action Γ as the Legendre transform of $W[J]$:

$$i\Gamma[\varphi_{cl}] \equiv -iW[J] - i\int J\varphi_{cl}$$

Prop As usual with Legendre-transform, we have

$$\frac{\delta}{\delta\varphi_{cl}}\Gamma = -J$$

Indeed, $\frac{\delta\Gamma[\varphi_{cl}]}{\delta\varphi_{cl}} = \frac{\delta}{\delta i\varphi_{cl}}\left(-iW - i\int J\varphi_{cl}\right)$

$$= -\int \frac{\delta J}{\delta\varphi_{cl}} \cdot \frac{\delta}{\delta iJ}(-iW) - \int \frac{\delta J}{\delta\varphi_{cl}} \cdot \underbrace{\frac{\delta}{\delta iJ}(-iW)}_{\equiv\varphi_{cl}} - J = -J$$

→ Let's understand what $\frac{\delta i\Gamma}{\delta i\varphi_{cl}}$ computes. We have:

p380 Peskin

$$\delta^4(x-y) = \frac{\delta}{\delta J(x)} J(y)$$

But we know that $-\frac{\delta i\Gamma}{\delta i\varphi_{cl}} = J$

$$= -\frac{\delta}{\delta J_x}\frac{\delta}{\delta\varphi_{cl}(y)}\Gamma = \int \frac{\delta\varphi_{cl}(z)}{\delta J(y)}\frac{\delta^2\Gamma}{\delta\varphi_{cl}(z)\,\delta\varphi_{cl}(x)} d^4z$$

$$= \int d^4z \, \frac{\delta^2 W}{\delta J(y)\,\delta J(z)} \, \frac{\delta^2\Gamma}{\delta\varphi_{cl}(z)\,\delta\varphi_{cl}(x)}$$

In a functional sense, this means:

$$\frac{\delta^2\Gamma}{\delta\varphi_{cl}\delta\varphi_{cl}} = \left(\frac{\delta^2 W}{\delta J\delta J}\right)^{-1} = D_{tot}^{-1} = k^2 - m^2 - \mathcal{M}(k^2, m^2, \lambda^2)$$

→ Higher order of functional derivative of Γ:

→ The general connected 3-pt function:

$$\frac{\delta^3(-iW)}{\delta iJ_x\,\delta iJ_y\,\delta iJ_z} = \frac{\delta}{\delta iJ_x}\left(\frac{\delta^2(-iW)}{\delta iJ_y\,\delta iJ_z}\right)$$

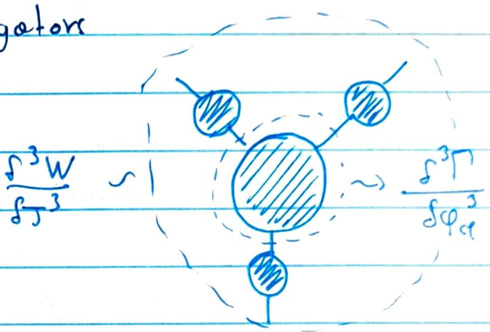
$$= \int \frac{\delta\varphi_{cl}}{\delta J}\cdot\frac{\delta}{\delta\varphi_{cl}}\left(\frac{\delta^2\Gamma}{\delta\varphi^{cl}_y\,\delta\varphi^{cl}_z}\right)^{-1}$$

"$\varphi = \frac{\delta W}{\delta J}$"

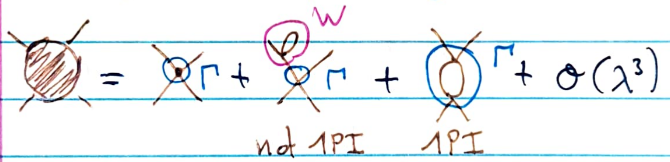
$$= \int \frac{\delta^2 W}{\delta J\,\delta J}\iint\left(\frac{\delta^2\Gamma}{\delta\varphi\,\delta\varphi}\right)^{-1}\frac{\delta^3\Gamma}{\delta\varphi\,\delta\varphi\,\delta\varphi}\left(\frac{\delta^2\Gamma}{\delta\varphi\,\delta\varphi}\right)^{-1}$$

$$= \iiint D\,D\,D\,\frac{\delta^3\Gamma}{\delta\varphi^{cl}\delta\varphi^{cl}\delta\varphi^{cl}}$$

↳ We see that $\delta^3\Gamma/\delta\varphi_{cl}^3$ gives the connected 3 pt-function stripped of 3 complete propagators



↳ Example, the $\lambda\varphi^4$ theory up to $\mathcal{O}(\lambda^2)$:



→ The effective action Γ is the object that contains the physically most interesting quantities (concerning quantum corrections) of a QFT.