CH2 GAUGE SYMMETRIES

2.1 Introduction

DEF A gauge symmetry is a symmetry of the action that depends on arbitrary parameters:

$$\delta_f \phi^i = R^i_\alpha[f^\alpha] \equiv R^i_\alpha f^\alpha + R_\alpha^{i\,\mu} \partial_\mu f^\alpha + \dots \quad \text{where } f^\alpha = f^\alpha(x, \phi)$$

$$\Rightarrow \delta_f \mathcal{L} = \partial_\mu k^\mu_f$$

⊙ Examples:

① QED: fields: $\phi^i = A_\mu, \bar{\Psi}, \psi$.

$$\to \mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \bar{\Psi} \gamma^\mu (\partial_\mu - e A_\mu) \psi + m \bar{\Psi} \psi$$

?

$$\to \delta_\epsilon \equiv i\epsilon \psi \rightsquigarrow \delta_\epsilon \bar{\Psi} = -\epsilon \bar{\Psi}, \ \delta_\epsilon A_\mu = \partial_\mu \epsilon \rightsquigarrow \delta_\epsilon \mathcal{L}_{QED} = 0$$

② YM:

$$\to \delta A^a_\mu = D_\mu \epsilon^a = \partial_\mu \epsilon^a + f^a_{bc} A^b_\mu \epsilon^c$$

③ GR

$$\to \delta g_{\mu\nu} = -\mathcal{L}_\xi g_{\mu\nu} = -(\xi^\rho \partial_\rho g_{\mu\nu} + \partial_\mu \xi^\rho g_{\rho\nu} + \partial_\nu \xi^\rho g_{\mu\rho})$$

Thm The Noether current for a gauge symmetry is trivial

DEMO From the definition, we have:

$$R^i_\alpha[f^\alpha] \frac{\delta L}{\delta \phi^i} = \left(R^i_\alpha f^\alpha + R^{i\,\mu}_\alpha \partial_\mu f^\alpha + \dots \right) \frac{\delta L}{\delta \phi^i}$$

$$= f^\alpha \left(R^i_\alpha \cdot \frac{\delta L}{\delta \phi^i} - \partial_\mu [R^{i\,\mu}_\alpha] \frac{\delta L}{\delta \phi^i} + \dots \right) + \partial_\mu \left[R^{i\,\mu}_\alpha \cdot f^\alpha \cdot \frac{\delta L}{\delta \phi^i} - \dots \right]$$

We define: $R^{+i}_\alpha \left[\frac{\delta L}{\delta \phi^i} \right] \equiv R^i_\alpha \cdot \frac{\delta L}{\delta \phi^i} - \partial_\mu [R^{i\,\mu}_\alpha] \frac{\delta L}{\delta \phi^i} + \dots$

and $S^\mu_f \equiv R^{i\,\mu}_\alpha \cdot f^\alpha \frac{\delta L}{\delta \phi^i} - \dots \propto$ EOM $\left(S^\mu_f \big|_{EOM} = 0 \right)$

Then: $R^i_\alpha[f^\alpha] \frac{\delta L}{\delta \phi^i} = f^\alpha R^{+i}_\alpha \left[\frac{\delta L}{\delta \phi^i} \right] + \partial_\mu S^\mu_f \left(\frac{\delta L}{\delta \phi^i} \right)$

Since $\delta_f \phi^i = R^i_\alpha[f^\alpha]$ is a symmetry $\forall f$, there exists a Noether current constructed as above: $R^i_\alpha[f^\alpha] \frac{\delta L}{\delta \phi^i} = \partial_\mu j^\mu_f$

Indeed, $\partial_\mu j_f^\mu = f^\alpha R^{+i}{}_\alpha \left[\frac{\delta L}{\delta \phi^i}\right] + \partial_\mu S_f^\mu\left(\frac{\delta L}{\delta \phi^i}\right)$

Since f^α is arbitrary, it can be taken as an independant new field and we can apply $\frac{\delta}{\delta f^\alpha}$ to find the Noether identities

$$R^{+i}{}_\alpha \left[\frac{\delta L}{\delta \phi^i}\right] = 0 \iff \partial_\mu (j_f^\mu - S_f^\mu) = 0$$

The Noether current is $j_f^\mu = k_f^\mu - \frac{\partial L}{\partial \phi^i_\mu} \delta_f \phi^i$

2.2 Mathematical tools

DEF Let $\mathcal{M}$ be a D-dimensional manifold. A p-form ($p \leq D$) is expressed as

$$\omega_p \equiv \frac{1}{p!} \omega_{\mu_1 \cdots \mu_p} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$$

The exterior derivative is defined as

$$d\omega \equiv \frac{1}{p!} \partial_\alpha \omega_{\mu_1 \cdots \mu_p} dx^\alpha \wedge dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$$

The Hodge star dual $*$ is defined through

$$* \omega_p \equiv \frac{1}{p!} \omega_{\mu_1 \cdots \mu_p} * dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$$

where $* dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p} \equiv \frac{1}{(D-p)!} \epsilon^{\mu_1 \cdots \mu_p}{}_{\mu_{p+1} \cdots \mu_D} dx^{\mu_{p+1}} \wedge \dots \wedge dx^{\mu_D}$

→ One gets: $d\omega = \frac{1}{(p+1)!} (d\omega)_{\alpha \mu_1 \cdots \mu_p} dx^\alpha \wedge dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}$

where $(d\omega)_{\alpha \mu_1 \cdots \mu_p} = (p+1) \partial_{[\alpha} \omega_{\mu_1 \cdots \mu_p]}$

Prop Let $\omega \in \Omega^p$. Then $d^2 \omega = 0$

DEF Let $\omega \in \Omega^p$. We say that ω is a closed form if $d\omega = 0$.
We say that ω is an exact form if

$\exists \eta$ such that $d\eta = \omega$ (notice $\eta \in \Omega^{p-1}$)

Prop Every exact form is necessarily closed since $d^2 = 0$

→ The cohomology try to find all η from $d\eta$

Lemma (Poincaré) Every closed p-form ω (s.t $d\omega = 0$), on an open ball in $\mathbb{R}^n$, is exact for $1 \leq p \leq n$:

$$\begin{cases} \omega \in \Omega^p \\ d\omega = 0 \end{cases} \Rightarrow \exists \eta \in \Omega^{p-1} \text{ s.t. } \omega = d\eta$$

$\boxed{\text{DEMO}}$

① If $p = 0$, the closed form can't be exact. Indeed,

$$\omega^0 = f(x) \; ; \; d\omega^0 = dx^\mu \frac{\partial f}{\partial x^\mu} = 0 \Rightarrow f = \text{cste}.$$

Wtf is happening? help

② If $p > 0$, we have: $d = dx^\mu \cdot \frac{\partial}{\partial x^\mu}$ and $\rho = x^\mu \frac{\partial}{\partial dx^\mu}$

→ We compute first:

$$\{d, \rho\}\omega = (d\rho + \rho d)\omega$$
$$= dx^\mu \frac{\partial}{\partial x^\mu}\left(x^\nu \frac{\partial}{\partial dx^\nu}\omega\right) + x^\mu \frac{\partial}{\partial dx^\mu}\left(dx^\nu \frac{\partial}{\partial x^\nu}\omega\right)$$
$$= \left\{dx^\mu \frac{\partial}{\partial dx^\mu} + x^\mu \frac{\partial}{\partial x^\mu}\right\}\omega^p$$

when $x^\mu \partial_\mu$ count the homogeneity

→ Let's show the Lemma:

$$\omega(x, dx) = \omega(0,0) + \int_0^1 dt \cdot \frac{d}{dt}\omega(tx, t\,dx)$$
$$= \omega(0,0) + \int_0^1 dt \left\{x^\mu \frac{\partial \omega}{\partial x^\mu}(tx, t\,dx) + dx^\mu \frac{\partial \omega}{\partial dx^\mu}(tx, t\,dx)\right\}$$
$$= \omega(0,0) + \int_0^1 dt \left\{\frac{1}{t}\left[\left(x^\mu \frac{\partial}{\partial x^\mu} + dx^\mu \frac{\partial}{\partial dx^\mu}\right)\omega\right](tx, t\,dx)\right\}$$
$$= \omega(0,0) + \int_0^1 \frac{dt}{t}\left[\{d,\rho\}\omega\right](tx, t\,dx)$$

but we know that $d\omega = 0$

$$= \omega(0,0) + \int_0^1 \frac{dt}{t}[\rho\omega](tx, t\,dx)$$
$$= \omega(0,0) + (d\eta)$$

∎

→ Example: $p = 1$. We have $d\omega^1 = 0 \Leftrightarrow \omega^1 = d\eta^0$.

Let $\omega^1 = g_\mu(x)\,dx^\mu$; $d\omega^1 = \partial_\nu g_\mu\, dx^\nu dx^\mu = \frac{1}{2}(\partial_\nu g_\mu - \partial_\mu g_\nu)\,dx^\nu dx^\mu$

Since $d\omega^1 = 0$, we have $\partial_\nu g_\mu = \partial_\mu g_\nu \Rightarrow g_\mu = \partial_\mu f$

Now, $\eta^0 = f(x)$ and $d\eta^0 = dx^\mu \partial_\mu f$

$$\Rightarrow \int_0^1 \frac{dt}{t}(\rho\omega^1)(tx, t\,dx) = \int_0^1 \frac{dt}{t}[x^\mu g_\mu](tx, t\,dx) = \int_0^1 dt\, x^\mu g(tx)$$

2.3 Noether current

→ We want to find the general solution of $\partial_\mu (j_f^\mu - S_f^\mu) = 0$

→ Let $(d^{n-p}x)_{\mu_1 \dots \mu_p} \equiv \frac{1}{p!} \frac{1}{(n-p)!} \epsilon_{\mu_1 \dots \mu_p \mu_{p+1} \dots \mu_n} dx^{\mu_{p+1}} \wedge \dots \wedge dx^{\mu_n}$

Simple example: $d^n x = \frac{1}{n!} \epsilon_{\mu_1 \dots \mu_n} dx^{\mu_1} \dots dx^{\mu_n} = dx^{\mu_1} \dots dx^{\mu_n}$

Then $d(\omega^{\mu_1 \dots \mu_p} (d^{n-p}x)_{\mu_1 \dots \mu_p}) = \omega^{\mu_1 \dots \mu_p}{}_{,\mu_p} (d^{n-p+1}x)_{\mu_1 \dots \mu_{p-1}}$

DEF | For a current j^μ, we define the $n-1$ form j:

$$\omega^{n-1} = j \equiv j^\mu (d^{n-1}x)_\mu$$
$$= j^1 dx^2 \dots dx^n - j^2 dx^1 dx^3 \dots dx^n + \dots + (-1)^n j^n dx^1 \dots dx^{n-1}$$

→ For example, $\omega^{n-p} = \omega^{\mu_1 \dots \mu_p} (d^{n-p}x)_{\mu_1 \dots \mu_p}$

$$d\omega^{n-p} = \omega^{\mu_1 \dots \mu_p}{}_{,\mu_p} (d^{n-p+1}x)_{\mu_1 \dots \mu_{p-1}}$$

→ Hence, in term of form, we find $dj = \partial_\mu j^\mu d^n x$ $(d^n x \equiv \mathrm{Vol})$

→ We now have: $d(j_f - S_f) = 0$. If we could use Poincaré lemma, then $j_f - S_f = d\sigma_f$ with $\sigma_f \in \Omega^{n-2}$ a $n-2$ form.

we can parametize this result:

$$j_f^\mu = S_f^\mu + \partial_\nu k_f^{[\mu\nu]}$$

→ We suppose now there exist a particular gauge parameter $\bar{f}^\alpha$ such that $R^i{}_\alpha[\bar{f}^\alpha] = 0$

Then $0 = R^i{}_\alpha[\bar{f}^\alpha] \frac{\delta \mathcal{L}}{\delta \phi^i} = \bar{f}^\alpha R^{+i}{}_\alpha \left[\frac{\delta \mathcal{L}}{\delta \phi^i}\right] + \partial_\mu S_{\bar{f}}^\mu \left(\frac{\delta \mathcal{L}}{\delta \phi^i}\right)$

$\Leftrightarrow \partial_\mu S_{\bar{f}}^\mu = 0$

↳ In p-form, it gives: $d S_{\bar{f}} = 0 \overset{\text{Poincaré}}{\Rightarrow} S_{\bar{f}} = dk_{\bar{f}} \approx 0$ ($= 0$ on EOM)

PROP | There exist a conserved $(n-2)$ form $k_{\bar{f}}$:

$$\partial_\nu k_{\bar{f}}^{[\mu\nu]} \approx 0 \text{ for all solution of the EOM.}$$

It can be built explicitly: $k_{\bar{f}} = \rho S_{\bar{f}}$

⊙ Example: gravity

→ Let $\bar\xi$ be a Killing vector for a generic metric:

$$R^i{}_\alpha[\bar f^\alpha] = 0 \Leftrightarrow \mathcal{L}_{\bar\xi}\, g_{\mu\nu} = 0$$

$$\Leftrightarrow \bar\xi^\rho \partial_\rho g_{\mu\nu} + \bar\xi^\rho{}_{,\mu}\, g_{\rho\nu} + \bar\xi^\rho{}_{,\nu}\, g_{\mu\rho} = 0$$

↳ Special case: flat metric. $g_{\alpha\beta} = \eta_{\alpha\beta}$. Then,

$$\mathcal{L}_{\bar\xi}\,\eta_{\mu\nu} = \bar\xi^\rho{}_{,\mu}\,\eta_{\rho\nu} + \bar\xi^\rho{}_{,\nu}\,\eta_{\mu\rho} = \bar\xi_{\nu,\mu} + \bar\xi_{\mu,\nu} = \bar\xi_{(\mu,\nu)} = 0$$

$\Rightarrow$ $\bar\xi^\alpha = a^\alpha + \omega^\alpha{}_\beta x^\beta$ with $\omega_{[\alpha\beta]} = \omega_{\alpha\beta}$

↳ Poincaré transformation!

For a generic metric, one can prove $dk^{n-2} \hat\approx 0$

$\Rightarrow$ $k^{n-2} \sim \cancel{f^{n-2}}\ 0 + d\eta^{n-2}$

↳ All conserved $n-2$ forms (on all solutions) are trivial

⊙ Example: semi-simple Yang-Mills theory.

→ We had $\delta_{\bar\epsilon} A_\mu^a = D_\mu \bar\epsilon^a = \partial_\mu \bar\epsilon^a + f^a_{bc} A_\mu^b \bar\epsilon^c = 0 \quad \forall A$

The solution will depend on the (Lie) algebra of the group.

ex: for $SU(2)$, the f^a_{bc} are the ϵ^{abc}.

→ Since it's true for all A, we take $A_\mu^a = 0$

$\Rightarrow$ $\bar\epsilon^a = cst$ and $f^a_{bc}\bar\epsilon^c = 0 \Rightarrow \bar\epsilon^c = 0$

↳ For a semi-simple Lie group, all $n-2$ form are trivial.

⊙ Example: electromagnetism.

→ We have $\partial_\mu \bar\epsilon = 0 \Longleftrightarrow \bar\epsilon = cst$

Let's compute the conserved $n-2$ form.

→ Consider $\mathcal{L} = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + A_\mu J^\mu$; $\partial_\mu J^\mu = 0$

↳ $\delta_\epsilon S = \delta_\epsilon \int d^4x\, \mathcal{L} \overset{!}{=} 0$; $\delta_\epsilon A_\mu = \partial_\mu \epsilon$ are gauge invariances.

↳ $\partial_\mu \epsilon \frac{\delta \mathcal{L}}{\delta A_\mu} = \partial_\mu(-\partial_\nu F^{\mu\nu} + J^\mu) = \epsilon\, \underbrace{\partial_\mu(-\partial_\nu F^{\mu\nu} + J^\mu)}_{=0} + \partial_\mu\big(\underbrace{\epsilon\{-\partial_\nu F^{\mu\nu} + J^\mu\}}_{=S_f^\mu}\big)$

↳ The current is conserved if $\epsilon = \bar\epsilon = cst$:

$S_{f}^\mu = -\partial_\nu(\bar\epsilon F^{\mu\nu})$ in a region where $J^\mu(x) = 0$

→ $k_{\bar\epsilon}^{\mu\nu} = F^{\mu\nu}$

→ $Q = \int_{t=cst} d\sigma_i F^{0i} = \int d\sigma_i E^i$ ← Gauss law

2.4 Linearized gauge theories

2.4.1 For an interacting gauge theory

→ We consider an action invariant under gauge symmetries:

$$S[\varphi] = \int d^n x \, \mathcal{L}[\phi^i, \phi^i_{,\mu}]$$

such that $\delta_f \phi^i = (R^i{}_\alpha[\phi])[f^\alpha] \Rightarrow R^i{}_\alpha[f^\alpha] \frac{\delta \mathcal{L}}{\delta \phi^i} = \partial_\mu S^\mu_f$

$\Leftrightarrow \delta_f \mathcal{L} = \partial_\mu k^\mu_f$

→ We want to expand the theory around a solution of the EOM:

$\phi^i = \Phi^i + \varphi^i$ such that $\frac{\delta \mathcal{L}}{\delta \phi^i}[\Phi] = 0$

Developping the action, we get:

$$S[\phi] = S[\Phi] + \int d^n x \left\{ \varphi^i \frac{\partial \mathcal{L}}{\partial \phi^i}\Big|_\Phi + \varphi^i_{,\mu} \frac{\partial \mathcal{L}}{\partial \phi^i_{,\mu}}\Big|_\Phi + \dots \right\}$$

$$+ \frac{1}{2} \int d^n x \left\{ \varphi^i \frac{\partial \mathcal{L}}{\partial \phi^i}\Big|_\Phi + \varphi^i_{,\mu} \frac{\partial \mathcal{L}}{\partial \phi^i_{,\mu}}\Big|_\Phi + \dots \right\} \left\{ \varphi^j \frac{\partial \mathcal{L}}{\partial \phi^j}\Big|_\Phi + \varphi^j_{,\mu} \frac{\partial \mathcal{L}}{\partial \phi^j_{,\mu}}\Big|_\Phi + \dots \right\}$$

Not | We adopt the multi index notation:

$$\partial_{(\mu)} \varphi^i \frac{\partial \mathcal{L}}{\partial \phi^i_{(\mu)}} \equiv \varphi^i \frac{\partial \mathcal{L}}{\partial \phi^i} + \varphi^i_{,\mu} \frac{\partial \mathcal{L}}{\partial \phi^i_{,\mu}} + \varphi^i_{,\mu} \varphi^j_{,\nu} \frac{\partial \mathcal{L}}{\partial \phi^i_{,\mu} \partial \phi^j_{,\nu}} + \dots$$

DEF | We define $\mathcal{L}^{(2)}$ as $\mathcal{L}^{(2)}[\varphi, \Phi] \equiv \partial_{(\mu)} \varphi^i \partial_{(\nu)} \varphi^j \frac{\partial^2 \mathcal{L}}{\partial \phi^i_{(\mu)} \partial \phi^j_{(\nu)}}$

Prop | We can write the variation as

$$\delta = \varphi^i_{(\mu)} \frac{\partial}{\partial \phi^i_{(\mu)}}$$

→ We then have:

$$S[\phi] = S[\Phi] + \int d^n x \, \varphi^i_{(\mu)} \cancel{\frac{\partial \mathcal{L}}{\partial \phi^i_{(\mu)}}\Big|_\Phi}^{\,0} + \frac{1}{2} \int d^n x \, \varphi^i_{(\mu)} \varphi^j_{(\nu)} \frac{\partial^2 \mathcal{L}}{\phi^i_{(\mu)} \phi^j_{(\nu)}}$$

$$= S[\Phi] + \frac{1}{2} \int d^n x \, \mathcal{L}^{(2)}[\varphi, \Phi]$$

Prop | Linearized equations of motion derive from quadratic piece of action: expansion of action around a solution has no linear term:

$$\delta \frac{\delta \mathcal{L}}{\delta \phi^j} = \frac{\delta}{\delta \varphi^j} \mathcal{L}^{(2)}[\varphi, \Phi]$$

DEMO We want to show that $\left(\delta \frac{\delta \mathcal{L}}{\delta \phi^i}\right)\Big|_{\Phi} = \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i}$

In general, $\frac{\delta \mathcal{L}}{\delta \phi^i} = \frac{\partial \mathcal{L}}{\partial \phi^i} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial \phi^i_\mu}\right) + \dots \equiv (-\partial)_{(\mu)} \frac{\partial \mathcal{L}}{\partial \phi^i_{(\mu)}}$

We have: $\delta \frac{\delta \mathcal{L}}{\delta \phi^i} = \delta (-\partial)_{(\nu)} \frac{\partial \mathcal{L}}{\partial \phi^j_{(\nu)}} = (-\partial)_{(\nu)} \left(\delta \frac{\partial \mathcal{L}}{\partial \phi^j_{(\nu)}}\right)$

$$= (-\partial)_{(\nu)} \left(\varphi^i_{(\mu)} \frac{\partial^2 \mathcal{L}}{\partial \phi^i_{(\mu)} \partial \phi^j_{(\nu)}}\right) = \frac{\delta}{\delta \varphi^j} \mathcal{L}^{(2)}[\varphi, \Phi]$$

- Expansion of the gauge symmetry:

→ We want to expand $R^i_\alpha[f^\alpha] \frac{\delta \mathcal{L}}{\delta \phi^i} = \partial_\mu S^\mu_f\left(\frac{\delta \mathcal{L}}{\delta \phi}\right)$ around Φ :

↳ $S^\mu_f\left(\frac{\delta \mathcal{L}}{\delta \phi}\right) = S^{\mu i(\nu)}_f \partial_{(\nu)} \frac{\delta \mathcal{L}}{\delta \phi^i}$

↳ $(R^i_\alpha[\Phi])[f^\alpha] \frac{\delta \mathcal{L}}{\delta \phi^i}\Big|_\Phi = \partial_\mu S^\mu_f\left(\frac{\delta \mathcal{L}}{\delta \phi}\Big|_\Phi\right) \Leftrightarrow 0 = 0$ not interesting.

→ $\delta R^i_\alpha[f^\alpha] \cdot \frac{\delta \mathcal{L}}{\delta \phi^i} + R^i_\alpha[f^\alpha] \delta . \frac{\delta \mathcal{L}}{\delta \phi^i} = \partial_\mu \delta S^{\mu i(\nu)}_f \partial_{(\nu)} \frac{\delta \mathcal{L}}{\delta \phi^i} + \partial_\mu S^{\mu i(\nu)}_f \partial_{(\nu)} \delta \frac{\delta \mathcal{L}}{\delta \phi^i}$

evaluated in Φ, we get

$$R^i_\alpha[f^\alpha]\Big|_\Phi \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i} = \partial_\mu S^\mu_f\left(\frac{\delta \mathcal{L}^{(2)}}{\delta \varphi}\right)\Big|_\Phi$$

Prop We showed that $R^i_\alpha[f^\alpha]\Big|_\Phi$ defines a gauge symmetry of the linearized theory.

→ $\delta_f \varphi^i = R^i_\alpha\big|_\Phi [f^\alpha]$ is a symmetry of the action and doesn't depend on φ

→ Second order expansion:

$$R^i_\alpha[f^\alpha]^{(0)}\Big|_f \left(\frac{\delta \mathcal{L}}{\delta \phi^i}\right)^{(2)} + R^i_\alpha[f^\alpha]^{(1)} \underbrace{\left(\frac{\delta \mathcal{L}}{\delta \phi^i}\right)^{(1)}}_{= \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i}} + R^i_\alpha[f^\alpha]^{(2)} \left(\frac{\delta \mathcal{L}}{\delta \phi^i}\right)^{(0)}\Bigg|_\Phi = \partial_\mu S^{\mu\,(2)}_f \Bigg|_{\substack{\Phi \\ f^\alpha}}$$

$$\Leftrightarrow (R^i_\alpha[f^\alpha])^{(1)} \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i} = \partial_\mu\left(S^{\mu(2)}_f\Big|_{\substack{\Phi \\ f}}\right)$$

DEF Were f^α are the reducibility parameters of the linearised theory

Prop. We showed that in linearized theory, the reducibility parameters define global symmetries ($d-1$ form conserved), on top of the usual Noether current of the full theory.

$$R^i{}_\alpha[\bar{f}^\alpha]^{(1)} \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i} = \partial_\mu \left(S_f^\mu \left(\frac{\delta \mathcal{L}}{\delta \phi^i} \right)^{(2)} \right) \Big|_{\bar{f}}$$

Remember $Q^i \frac{\delta \mathcal{L}^{(2)}}{\delta \varphi^i} = \partial_\mu j^\mu$

⊙ Example: GR

→ We set $\phi^i = g_{\mu\nu}$ and expand $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

↳ $\delta_f \phi^i = R^i{}_\alpha[f^\alpha]$ gives $\delta_\xi g_{\mu\nu} = \mathcal{L}_\xi g_{\mu\nu}$

↳ We see that $\delta_\xi h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \mathcal{L}_\xi \eta_{\mu\nu}$

⇒ There exist a solution to the equation $R^i{}_\alpha[\bar{f}^\alpha] = 0$

What's the point? symmetry of the solution gave a symmetry of the lin. theory?